Identifying Optimal Parameters for Approximate Randomized Algorithms

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WAX 2019
June 22, 2019, Phoenix, Arizona
Randomized Approximate Algorithms

Modern applications deal with large amounts of data

Obtaining exact answers for such applications is resource intensive

Randomized Approximate algorithms give a “good enough” answer in a much more efficient manner
Randomized Approximate Algorithms

Used in many domains

- HyperLogLog, Bloom filter - Data analytics
- Approximate matrix multiplication - Numerical linear algebra
- Locality sensitive hashing – Fingerprinting multimedia

Often sub-linear in space/ runtime

Come with analytically derived specifications of accuracy/performance.
- e.g., an algorithm will have small errors with high probability
Randomized Approximate Algorithms

Randomized approximate algorithms have attracted the attention of many authors and researchers.

Developers struggle to properly test/optimize implementations of these algorithms.
Example: Count-min Sketch

• Count the **frequency of unique elements** in a large data set using sub-linear space

• Provides an estimate of the frequency with a bounded error

Data set: \[x, y, z, x, \ldots, x]\n
<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>:</td>
<td>95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y)</td>
<td>:</td>
<td>135</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(z)</td>
<td>:</td>
<td>935</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example: Count-min Sketch

• Count the frequency of unique elements in a data set using sub-linear space
• Use \((h \times w)\) counters

\[
\begin{array}{cccccccc}
\text{hash}_1 & 0 & 0 & 0 & 0 & \ldots & 0 \\
\text{hash}_2 & 0 & 0 & 0 & 0 & \ldots & 0 \\
\ldots & 0 & 0 & 0 & 0 & \ldots & 0 \\
\text{hash}_h & 0 & 0 & 0 & 0 & \ldots & 0 \\
\end{array}
\]
Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

Data set: $\mathbf{x}$

<table>
<thead>
<tr>
<th></th>
<th>hash$_1$</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>...</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>hash$_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>hash$_h$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
</tbody>
</table>
Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

Data set:  

<table>
<thead>
<tr>
<th>hash_1</th>
<th>0</th>
<th>0+1</th>
<th>0</th>
<th>0</th>
<th>...</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>hash_2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0+1</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>hash_n</td>
<td>0+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
</tbody>
</table>
Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

Data set: \[ x \quad y \]

<table>
<thead>
<tr>
<th>hash_1</th>
<th>0+1</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>...</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>hash_2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1+1</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>hash_n</td>
<td>1</td>
<td>0</td>
<td>0+1</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
</tbody>
</table>
Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

Data set: \(x\ y\ z\ x\ ...\ x\)

<table>
<thead>
<tr>
<th>hash_1</th>
<th>50</th>
<th>200</th>
<th>12</th>
<th>454</th>
<th>...</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>hash_2</td>
<td>12</td>
<td>213</td>
<td>21</td>
<td>132</td>
<td>...</td>
<td>7657</td>
</tr>
<tr>
<td>...</td>
<td>49</td>
<td>842</td>
<td>12</td>
<td>23</td>
<td>...</td>
<td>67</td>
</tr>
<tr>
<td>hash_h</td>
<td>343</td>
<td>5</td>
<td>121</td>
<td>23</td>
<td>...</td>
<td>435</td>
</tr>
</tbody>
</table>
Example: Count-min Sketch

• Count the frequency of unique elements in a data set using sub-linear space

Data set: \[ x \, y \, z \, x \, ... \, y \]

| \( \text{hash}_1 \) | 50 | 200 | 12 | 454 | ... | 64 |
| \( \text{hash}_2 \) | 12 | 213 | 21 | 132 | ... | 7657 |
| ... | 49 | 842 | 12 | 23 | ... | 67 |
| \( \text{hash}_n \) | 343 | 5 | 121 | 23 | ... | 435 |

Query: \[ x \]
Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

Data set: \[ \text{x y z x ... y} \]

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<td>23</td>
<td>...</td>
<td>435</td>
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Query: \[ \text{x} \]

Estimate count: \( \min(343, 200, 132, ...) = 132 \)
Count-min Sketch Accuracy Specification*

Correctness Guarantee:

\[ P[\text{error} < N \cdot \epsilon] > 1 - \delta \]

- \text{error} — difference between estimate and actual count
- \( N \) — size of the data set
- Number of hash functions (h) and the number of bins per hash (w) is set using the values for \( \epsilon \) and \( \delta \)

\[ w = \lceil e/\epsilon \rceil, \quad h = \lceil \ln(1/\delta) \rceil \]

AxProf: Algorithmic Profiling for Randomized Approximate Programs*

Tests if the implementations satisfies the algorithm’s specifications

The specification provided in a formal notation

• Generate inputs according to different distribution
• Gather samples and aggregate data
• Select appropriate statistical test

AxProf : Count-min Sketch Accuracy Testing

**Math Specification:** \( P [\text{error} < N \times \epsilon] > 1 - \delta \)

**AxProf specification:**

Input list of (list of real);
Output list of (list of int);
forall i in unique(Input)
Probability over runs

\[ \text{error}(i, \text{Input}, \text{Output}) < |\text{Input}| \times \text{epsilon} \] > 1 - delta

AxProf: **PASS/FAIL**
Setting algorithm parameters

How to set number of hash functions (h) and number of bins (w)?

Analytical specification:

\[ P[ \text{error} < N \cdot \epsilon] > 1 - \delta \]

To achieve this guarantee with minimum memory usage

\[ w = \lceil 2.718 / \epsilon \rceil, \quad h = \lceil \ln(1 / \delta) \rceil \]

Example:

\[ P[\text{error} < 5000] > 0.99 \Rightarrow w=534, \ h=5 \]

Observed errors for a randomly generated dataset in an implementation of Count-min
Analytical Error Guarantees Are Conservative

Take into account worst-case scenarios or perform average case analysis for a large input domain

Algorithm implementers can implement different behavior than specified
• Use of polyalgorithms
• Allocate more resources than required (Eg: Bigger arrays)

For some applications it is not possible to derive analytical models due to complex interactions among parameters (e.g., SFFT)

Alternative: Build empirical models to identify algorithm parameter values that satisfy a user's accuracy requirements while optimally utilizing resources (And satisfying the analytical accuracy guarantee)
Setting algorithm parameters: Count-min

Observed errors for a randomly generated dataset in an implementation of Count-min
Setting algorithm parameters: Count-min
Count-min Sketch Empirical Model of Accuracy

• Identify a representative input set – Lists of integers drawn from Zipf distributions (using AxProf input generator)

• Identify the possible configurations – the ranges of tunable parameters - e.g., w:[1-1000], h:[1-10]

• A tuning objective for each algorithm - optimize memory usage

• We used OpenTuner to identify optimal parameter values for error thresholds that also satisfy the analytical specification
Building Empirical Models

For each accuracy threshold

- Start with the default parameters (from analytical model)

  - Use Autotuning to search for better algorithm parameters

    - If not
      - Confirm that the analytical guarantee is still satisfied with AxProf

    - If found
      - Record
Building Empirical Models

For each accuracy threshold

If not

Start with the default parameters (from analytical model)

Use Autotuning to search for better algorithm parameters

If found

Confirm that the analytical guarantee is still satisfied with AxProf

Record

If not

Used an input generator from AxProf to generate random datasets
Setting Algorithm Parameters: Count-min

The algorithm finishes 30% faster and uses 50% less memory

\( P[\text{error} < 5000] > 0.99 \Rightarrow w=396, h=3 \)

Observed errors for a randomly generated dataset in an implementation of Count-min
Benefits of an Empirical(tuned) Model

For the specification:
\[ P[\text{error} < N \times \text{Epsilon}] > 1 - \delta \]
Future Directions: Adaptive Algorithms

• When the algorithm accuracy is data dependent - algorithm parameters can be set based on the input to achieve optimal performance

• However, in many of the algorithms the input is very large, therefore pre-analyzing the data may not be possible

• Requires low cost initial analysis of huge data sets
Future Directions: Runtime Monitoring

• An implementation can periodically estimate the error at runtime
• If that estimate starts to exceed an acceptable threshold, issue warnings or change to a more accurate configuration of the algorithm
• Error estimates need to be low cost for benefits
Future Directions: Comparing Implementations

• Two implementations that satisfy the same analytical specifications of the algorithm can have widely varying behavior
• Optimal behavior can be used to compare implementations of the same algorithm
Conclusion

Randomized approximate algorithms have conservative analytical probabilistic accuracy specification

Hybrid models of accuracy/performance can provide better resource usage while providing similar guarantee

Future directions
• Adaptive algorithms
• Runtime monitoring
• Reducing offline training time
• Selecting among multiple implementations