Identifying Optimal Parameters for Approximate Randomized Algorithms

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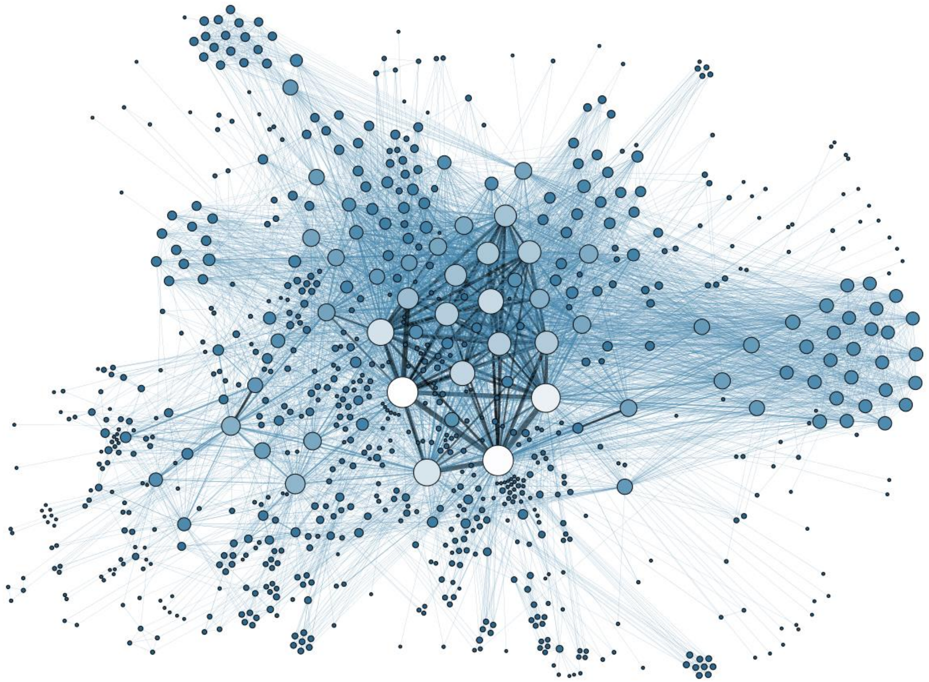
Randomized Approximate Algorithms

Modern applications deal with large amounts of data

Obtaining exact answers for such applications is resource intensive

Randomized Approximate algorithms give a "good enough" answer in a much more efficient manner



Randomized Approximate Algorithms

Used in many domains

- HyperLogLog, Bloom filter - Data analytics
- Approximate matrix multiplication - Numerical linear algebra
- Locality sensitive hashing – Fingerpriting multimedia

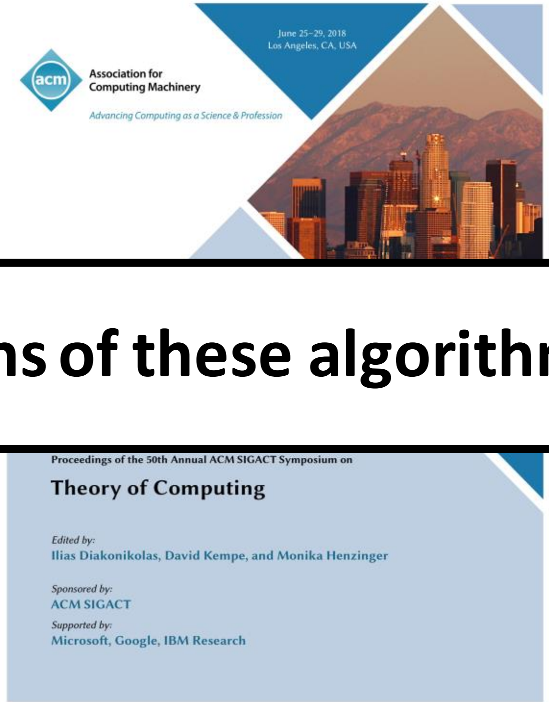
Often sub-linear in space/ runtime

Come with analytically derived specifications of accuracy/performance.

- e.g., an algorithm will have small errors with high probability

Randomized Approximate Algorithms

Randomized approximate algorithms have attracted the attention of many authors and researchers



Developers struggle to properly test/optimize implementations of these algorithms

Example: Count-min Sketch

- Count the **frequency of unique elements** in a large data set using sub-linear space
- Provides an estimate of the frequency with a bounded error

Data set: **x** **y** **z** **x** **…** **x**

x	:	95
y	:	135
z	:	935

Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space
- Use (h * w) counters

$hash_1$	0	0	0	0	…	0
$hash_2$	0	0	0	0	…	0
…	0	0	0	0	…	0
$hash_h$	0	0	0	0	…	0

h

w

Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

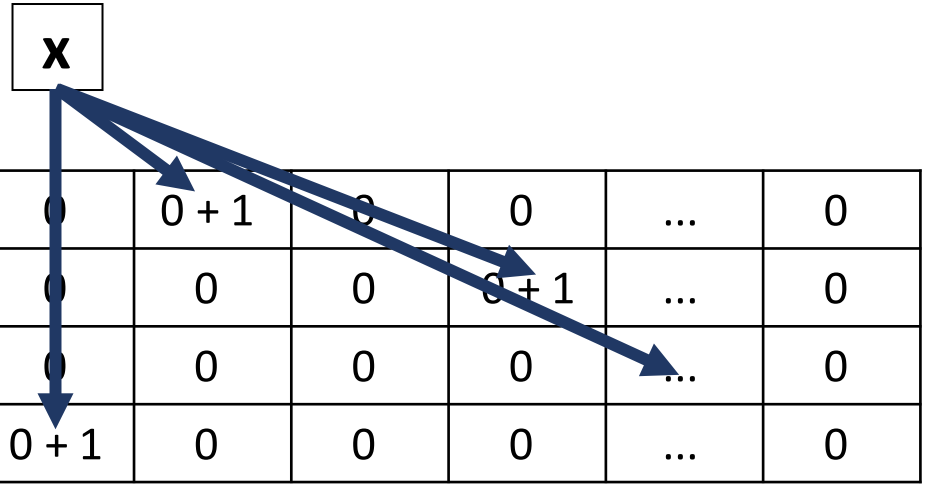
Data set: **x**

$hash_1$	0	0	0	0	...	0
$hash_2$	0	0	0	0	...	0
...	0	0	0	0	...	0
$hash_h$	0	0	0	0	...	0

Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

Data set: **x**



$hash_1$	0	0 + 1	0	0	...	0
$hash_2$	0	0	0	0 + 1	...	0
...	0	0	0	0	...	0
$hash_h$	0 + 1	0	0	0	...	0

Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

Data set: **x** **y**

$hash_1$	0 + 1	1	0	0	...	0
$hash_2$	0	0	0	1 + 1	...	0
...	0	0	0	0	...	0
$hash_h$	1	0	0 + 1	0	...	0

Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

Data set: | **x** | **y** | **z** | **x** | **…** | **x** |

$hash_1$	50	200	12	454	…	64
$hash_2$	12	213	21	132	…	7657
…	49	842	12	23	…	67
$hash_h$	343	5	121	23	…	435

Example: Count-min Sketch

- Count the frequency of unique elements in a data set using sub-linear space

Data set: **x** **y** **z** **x** **…** **y**

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Query: **x**

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Query: x

Estimate count: min(343, 200, 132, …) = 132

Count-min Sketch Accuracy Specification*

Correctness Guarantee:

$$P\,[\,error < N\ *\epsilon] > 1\ -\ \delta$$

- $error$ $-$ difference between estimate and actual count
- $N -$ size of the data set
- Number of hash functions (h) and the number of bins per hash (w) is set using the values for ϵ and δ

$$w = \lceil e/\epsilon \rceil,\ h = \lceil \ln(1/\delta) \rceil$$

*G. Cormode and S. Muthukrishnan, “An improved data stream summary: the Count-Min sketch and its applications,” Journal of Algorithms, vol. 55, 2005

AxProf: Algorithmic Profiling for Randomized Approximate Programs*

Tests if the **implementations** satisfies the algorithm’s specifications

The specification provided in a **formal notation**

- Generate inputs according to different distribution
- Gather samples and aggregate data
- Select appropriate statistical test

*Keyur Joshi, Vimuth Fernando, and Sasa Misailovic. 2019. Statistical algorithmic profiling for randomized approximate programs. (ICSE '19)

AxProf : Count-min Sketch Accuracy Testing

Math Specification: $P\,[\,error < N\; * \epsilon] > 1\; - \;\delta$

AxProf specification:

```
Input list of (list of real);
Output list of (list of int);
forall i in unique(Input)
Probability over runs
   [error(i, Input, Output) < |Input| * epsilon] > 1 - delta
```

AxProf: PASS/FAIL

Setting algorithm parameters

How to set number of hash functions (h) and number of bins (w)?

Analytical specification:

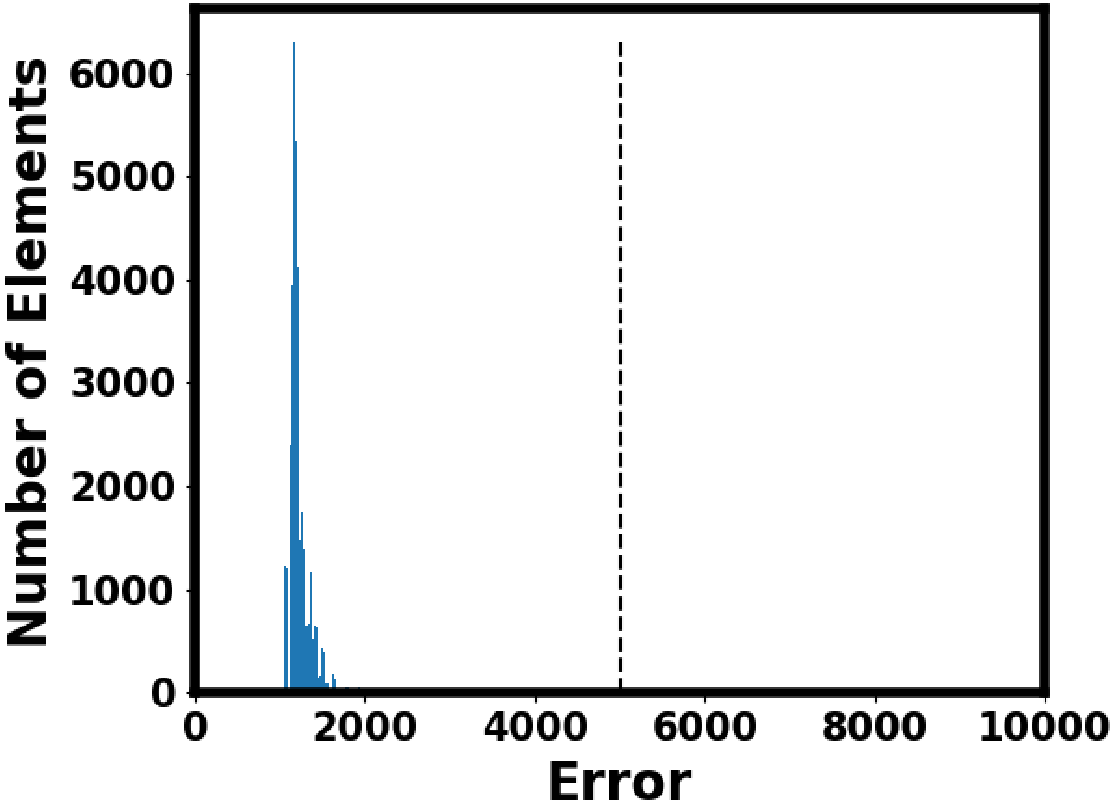
$$P[\text{ error} < N * \epsilon] > 1 - \delta$$

To achieve this guarantee with minimum memory usage

$$w = \lceil 2.718/\epsilon \rceil,\ h = \lceil \ln(1/\delta) \rceil$$

Example:

P[error < 5000] > 0.99 ⇒ w=534, h=5



Observed errors for a randomly generated dataset in an implementation of Count-min

Analytical Error Guarantees Are Conservative

Take into account worst-case scenarios or perform average case analysis for a large input domain

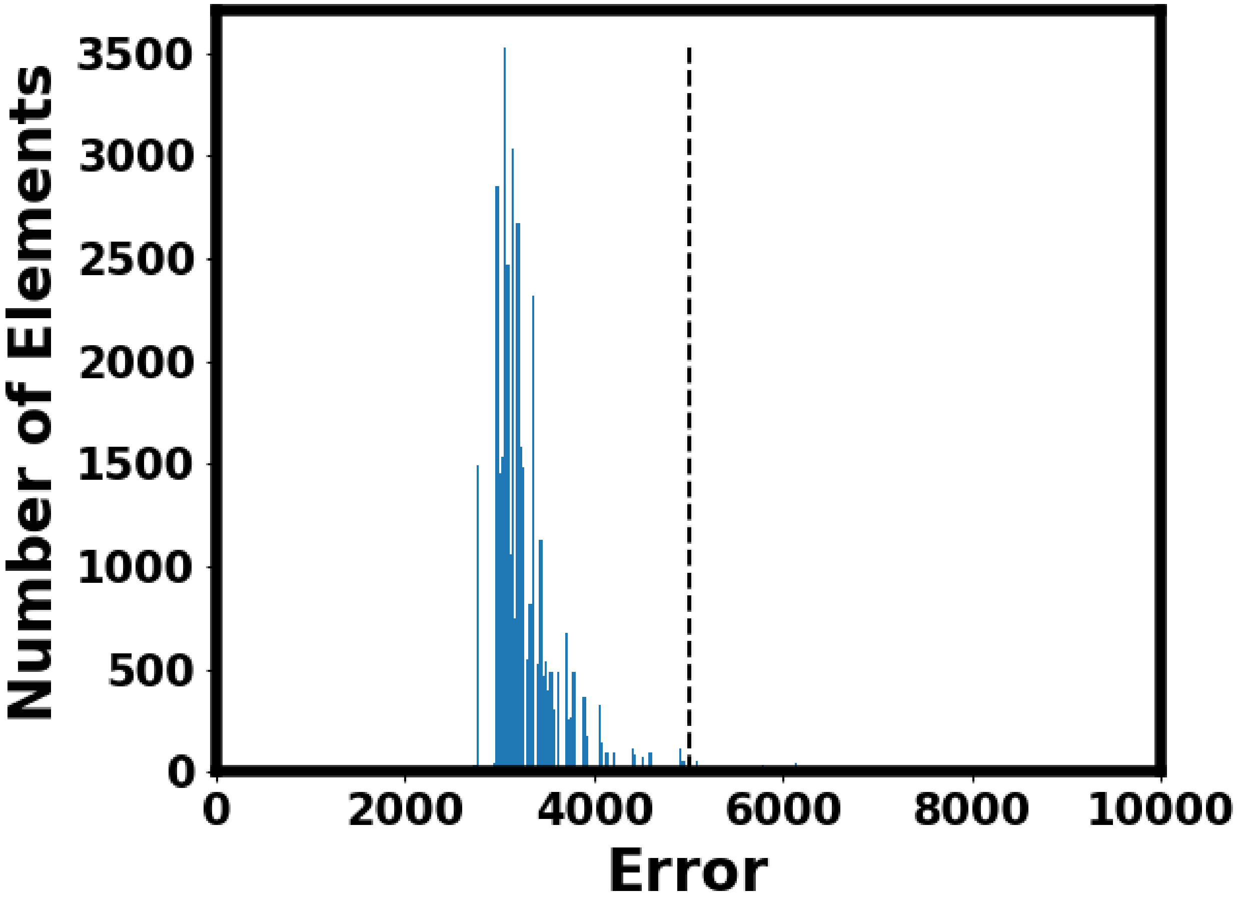
Algorithm implementers can implement different behavior than specified

- Use of polyalgorithms
- Allocate more resources than required (Eg: Bigger arrays)

For some applications it is not possible to derive analytical models due to complex interactions among parameters (e.g., SFFT)

Alternative: Build **empirical models** to identify algorithm parameter values that **satisfy a user's accuracy requirements** while **optimally utilizing resources** **(And satisfying the analytical accuracy guarantee)**

Setting algorithm parameters: Count-min



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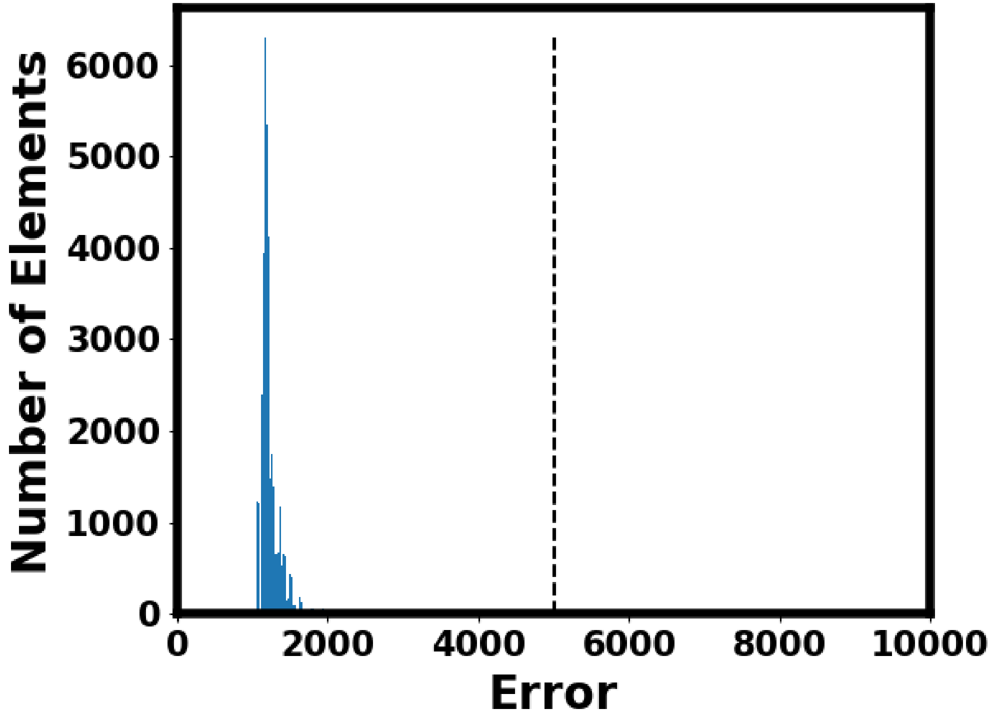
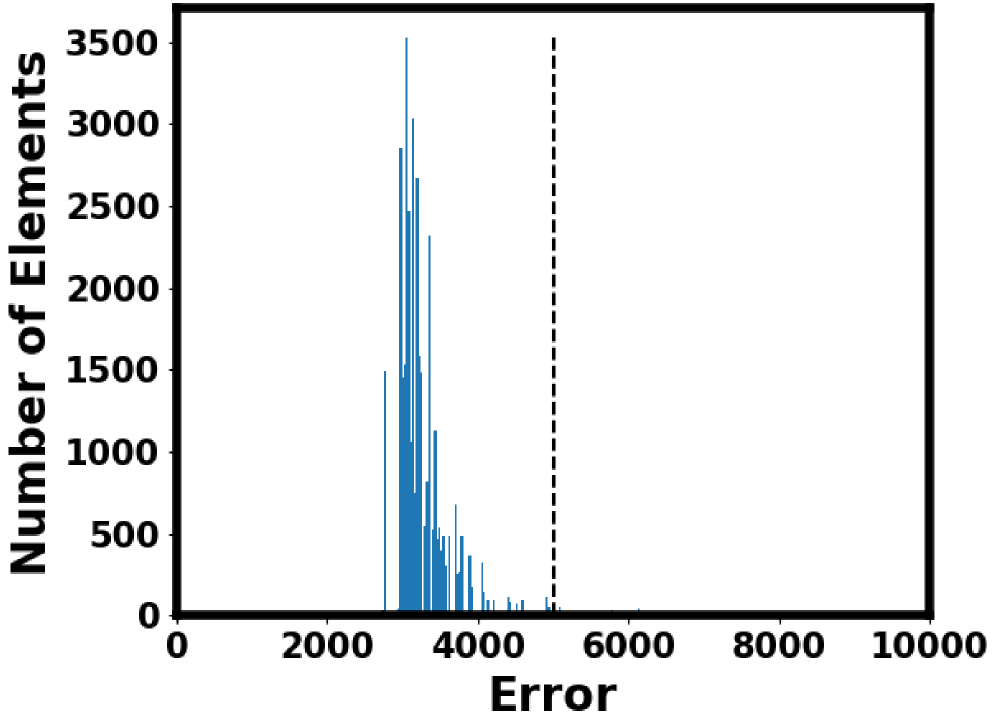
Setting algorithm parameters: Count-min



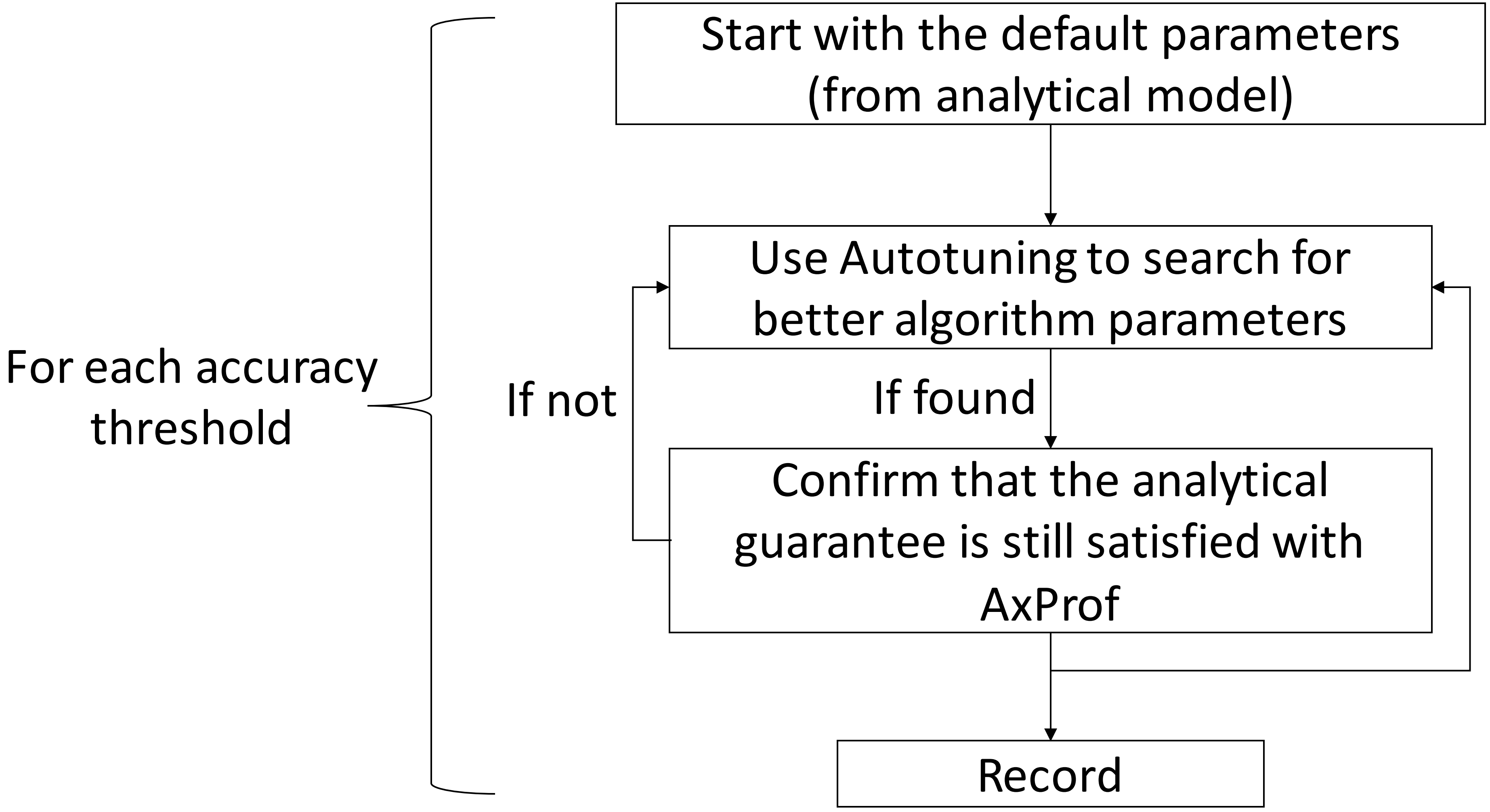


Count-min Sketch Empirical Model of Accuracy

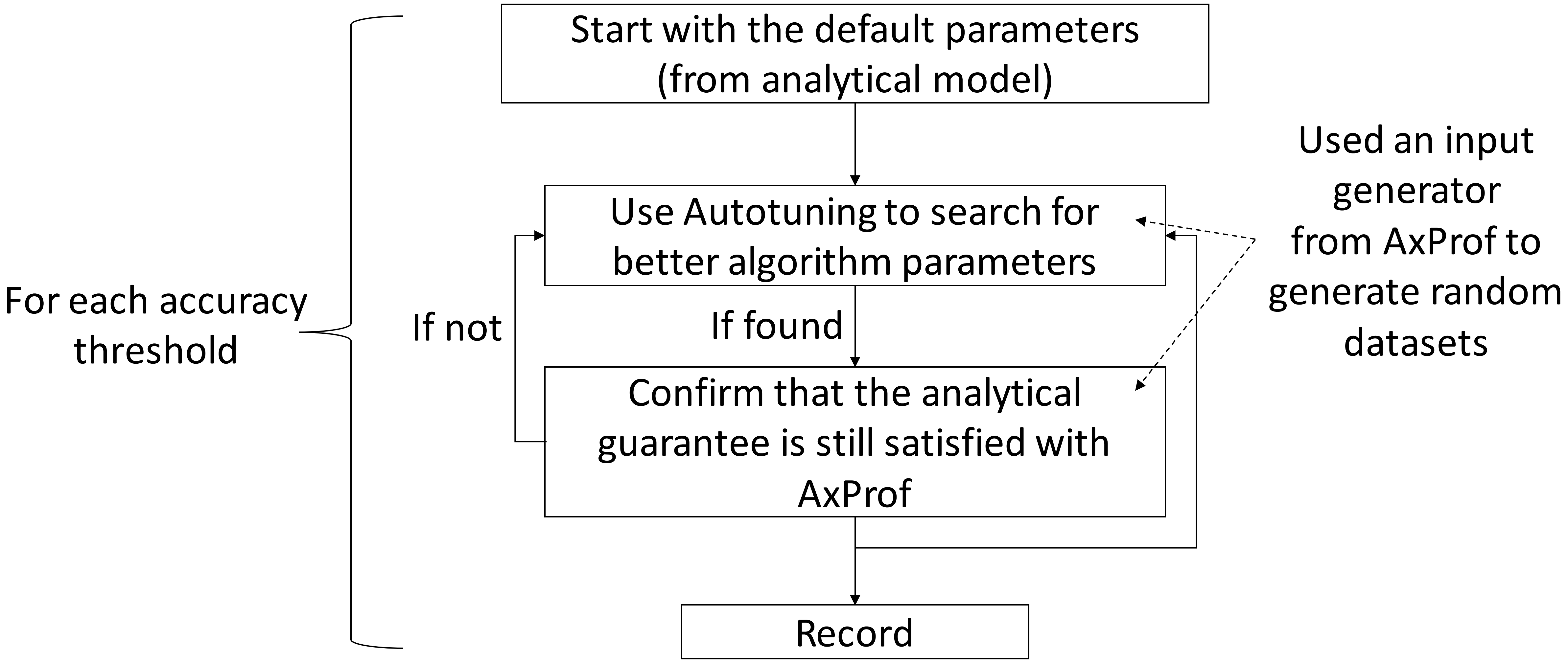
- Identify a representative input set – Lists of integers drawn from Zipf distributions (using AxProf input generator)
- Identify the possible configurations – the ranges of tunable parameters - e.g., w:[1-1000], h:[1-10]
- A tuning objective for each algorithm - optimize memory usage

- We used OpenTuner to identify optimal parameter values for error thresholds **that also satisfy the analytical specification**

Building Empirical Models



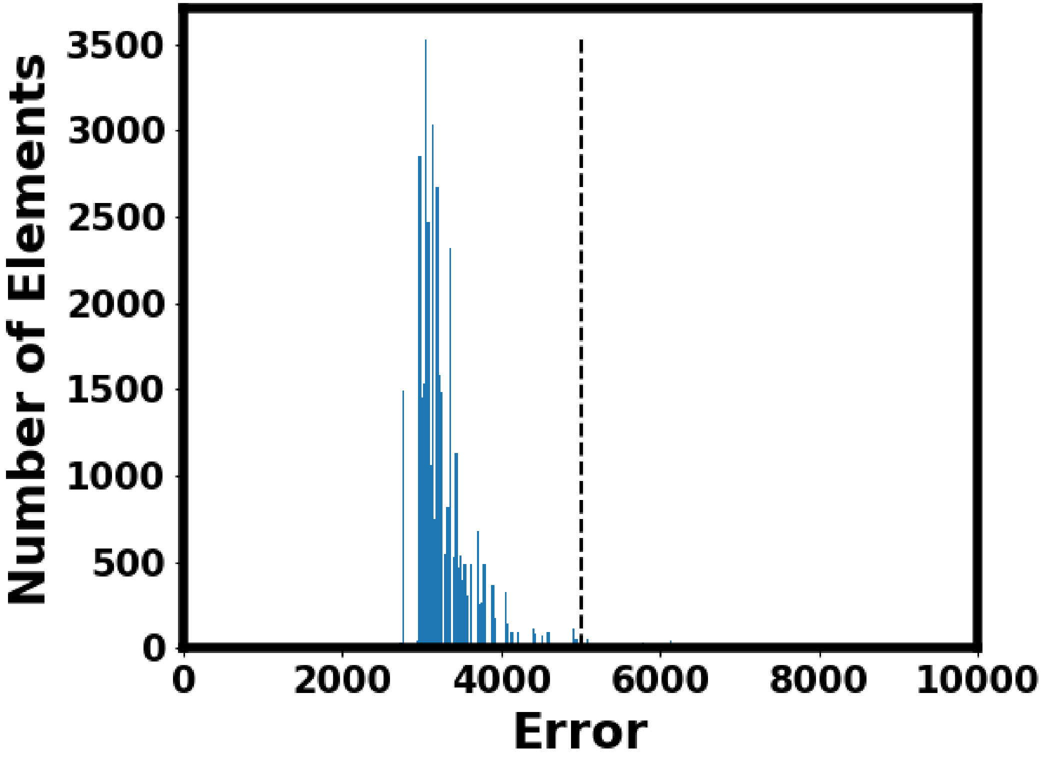
Building Empirical Models



Setting Algorithm Parameters: Count-min

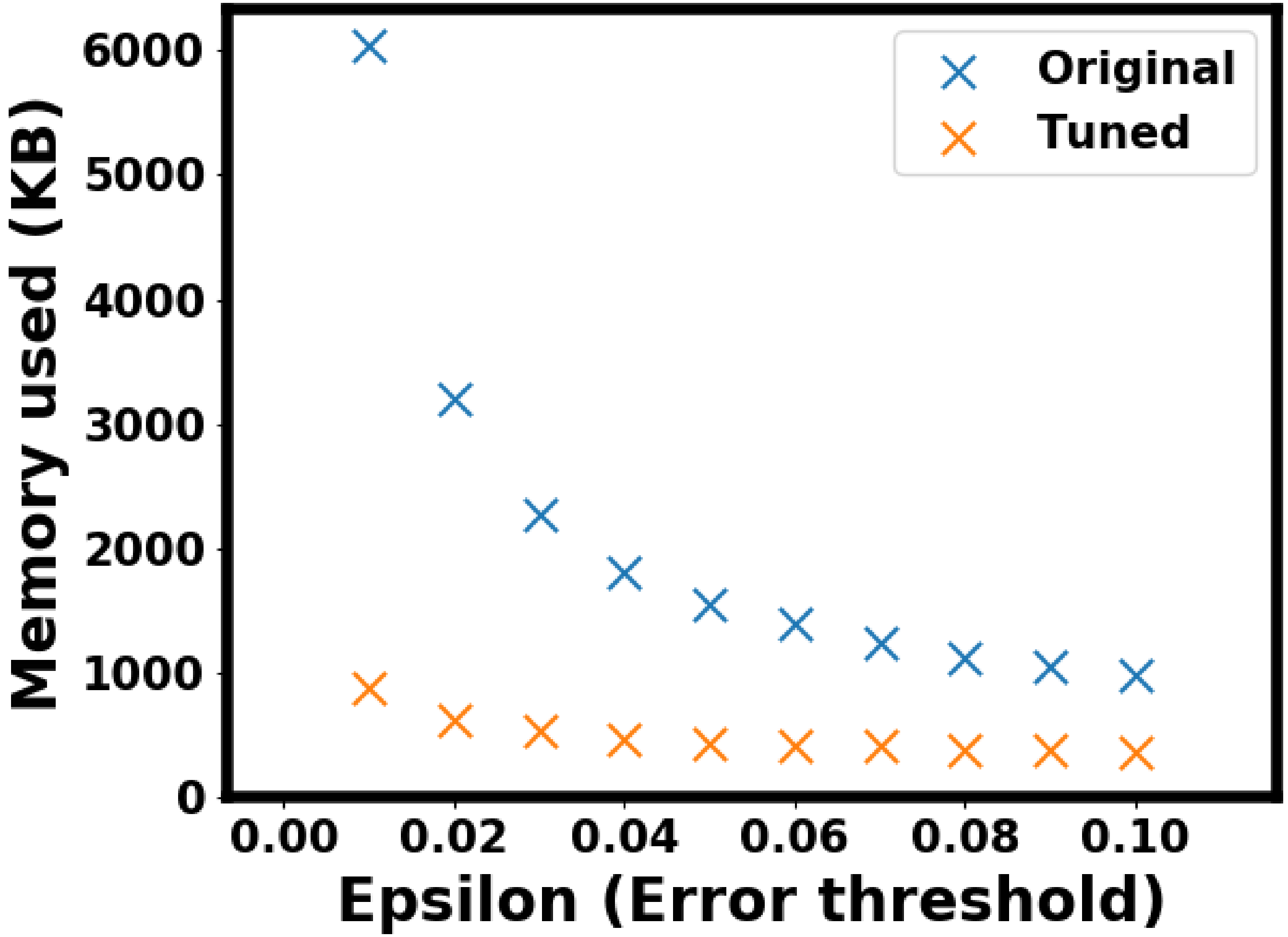
The algorithm finishes 30% faster and uses 50% less memory

(P[error < 5000] > 0.99 ⇒ w=396, h=3)



Observed errors for a randomly generated dataset in an implementation of Count-min

Benefits of an Empirical(tuned) Model



For the specification:

P[error < N * Epsilon] > 1 - δ

Future Directions: Adaptive Algorithms

- When the algorithm accuracy is data dependent - algorithm parameters can be set based on the input to achieve optimal performance
- However, in many of the algorithms the input is very large, therefore pre-analyzing the data may not be possible
- Requires low cost initial analysis of huge data sets

Future Directions: Runtime Monitoring

- An implementation can periodically estimate the error at runtime
- If that estimate starts to exceed an acceptable threshold, issue warnings or change to a more accurate configuration of the algorithm
- Error estimates need to be low cost for benefits

Future Directions: Comparing Implementations

- Two implementations that satisfy the same analytical specifications of the algorithm can have widely varying behavior
- Optimal behavior can be used to compare implementations of the same algorithm

Conclusion

Randomized approximate algorithms have conservative analytical probabilistic accuracy specification

Hybrid models of accuracy/performance can provide better resource usage while providing similar guarantee

Future directions

- Adaptive algorithms
- Runtime monitoring
- Reducing offline training time
- Selecting among multiple implementations